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State estimation using model order reduction for unstable systems

by

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State estimation using model order reduction for unstable systems

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[illegible]

a ca . *Da a a a* c a fi d b a b c b b a a da a a ca d . I c -d a a a a da a a (4D-Va) a b d a a a a b

$$\min_{x} \phi(x) = f(x)^T f(x); \tag{1}$$

c $f(x)$ c d a ca d T d b a a a a a Ga -N d, a a a c a 4D-Va [1, 2]. W a d a c a d d Jac b a c $f(x)$ a d c Jac b a a ca d c d a a a d (TLM). S fica ca d c d a d a d ab b a . I ab TLM a d a ca d a ab a fi a T a c b ab a a ab d b d .

A c d a ac d c c b (1) a a TLM b a a d d a a a a . W ad a a a ac ca c

I ac c , a a a b (1) f a d fi d (2) ca b d b a Ga -N d. T a a a a ac a (k) a a a c

$$\phi(\delta x_0^{(k)}) = \|J_f \delta x_0^{(k)} + f(x_0^{(k)})\|_2^2; \tag{3}$$

J_f d Jac b a f. T a d fi d a $x_0^{(k+1)} = x_0^{(k)} + \delta x_0^{(k)}$. I (2) a Jac b a J_f b

$$J_f = \left[\left(B_0^{\frac{1}{2}}\right)^T \quad \left(R_0^{\frac{1}{2}} H_0\right)^T \quad \left(R_1^{\frac{1}{2}} H_1 M_{1;0}\right)^T \quad \cdots \quad \left(R_N^{\frac{1}{2}} H_N M_{N;0}\right)^T \right]^T; \tag{4}$$

a d b a a d d a c $H_i := \frac{\partial h_i}{\partial x_i}(x^{($ $x^{($

F a a d (k) d . W
c d a - a a a a

$$\mathcal{S} : \begin{cases} \delta x_{i+1} &= M \delta x_i; \\ d_i &= H \delta x_i; \end{cases} \tag{6}$$

a (5), c a a c M a d H a a -
a - a a $M_{i+1;i}$ a d H_i d
$[t_0; t_N]$. T a a $\delta x_0 = B_0^{\frac{1}{2}} !$ a a d b d a d a -
ab a a d c a a c a $B_0 \in \mathbb{R}^{n \quad n}$, $! \sim \mathcal{N}(0; I)$.
T a c T (6) a c a a d
c d c b b a c d a . I
d fi d a

$$T : \mathbb{C} \rightarrow \mathbb{R}^{p \quad m}; \tag{7}$$

$$z \mapsto T(z) := H(zI - M)^{\ 1} B_0^{\frac{1}{2}}: \tag{8}$$

T (6) a ab , . . a
a M a d a a d c c . T b ab fi d
d a a (6) d a ab d d c c
ab . T d α-b d d ba c d ca a
c b d d [6] α-*b ded* , . . . a
a a M a c c a d a
ad α. F a ab $\mathcal{S}$ b fi d
c a α. T α-b d d ba c d ca c c
a d a a c U a d V, c , c a c d
,

$$\mathcal{S} : \begin{cases} \delta \hat{x}_{i+1} &= U^T M V \delta \hat{x}_i; \\ \hat{d}_i &= H V \delta \hat{x}_i; \end{cases} \tag{9}$$

d a c $\delta \hat{x}_i = U^T \delta x_i \in \mathbb{R}^r$, $r \ll n$, a a
d $\mathcal{S}$ M c , a ba b d
c $h_{1;}$ - [6]:

$$\|T - \hat{T}\|_{h_{\infty,\alpha}} \leq 2(\sigma^{(}$$

$F : \mathbb{C} \to \mathbb{R}^{m \quad p}$ a c c c c a d ad α a d, , α-b d d . T ca $\sigma^{(\ )}_{r+1}; : : : ; \sigma^{(\ )}_{n}$ a Ha a α-ca d $\mathcal{S}$, b a c

$$T \quad = \frac{1}{\alpha} H(zI - \frac{1}{\alpha} M) \quad {}^{1} B_0^{\frac{1}{2}};$$

d Ga -N c d . A a d c d a d a Ead d $x-z$ a d ba c c ab , c d a c a a d d .

4.1. E e e a de g

T d a a 2D Ead d [7] a d - c b d. T ba c a b a a a d a , z, a d a b d a b da , $z = \pm\frac{1}{2}$.

F [8] a d a a - c a c (QGPV) . T a a b ba b a c , $b = b(x; z; t)$, b da , $z = \pm\frac{1}{2}$, a $t = 0$. T d ca c d ba c a c- , $= (x; z; t)$, c a fi :

$$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} = 0; \qquad z \in [-\frac{1}{2}; \frac{1}{2}]; \; x \in [0; X]; \tag{13}$$

b da c d

$$\frac{\partial}{\partial z} = b; \qquad z = \pm\frac{1}{2}; \; x \in [0; X]: \tag{14}$$

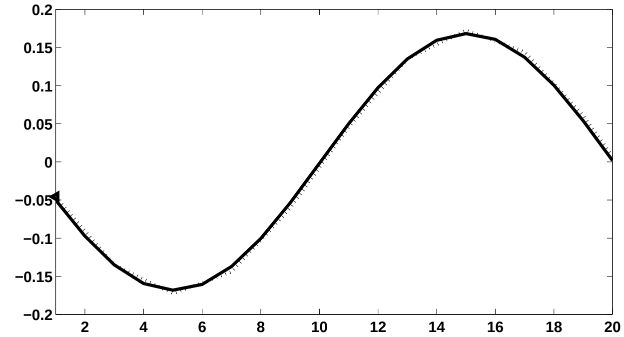
P ba ba c a a ad c d a b ba c a fl a d c b d b -d a QG d a c a :

$$\left(\frac{\partial}{\partial t} + z\frac{\partial}{\partial x}\right) b = \frac{\partial}{\partial x}; \qquad z = \pm\frac{1}{2}; \; x \in [0; X]: \tag{15}$$

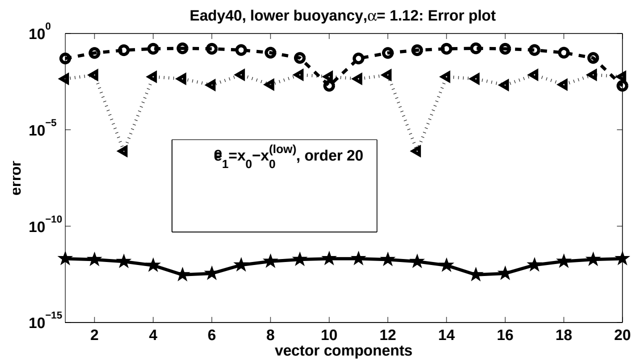
T a a b da c d x-d c a a b d c c a a a , t, a d , z, $b(0; z; t) = b(X; z; t)$ a d $(0; z; t) = (X; z; t)$. A [8] d a x, z, a d t.

I a d , Ead d d c d 11 ca a c . T a 20 d a 40 d d a c b. T ad c a a d c d a a ad c c . W [8] d a . T b a a H c c a b a a a b a c .

d c d 20, c c a a
b ab .



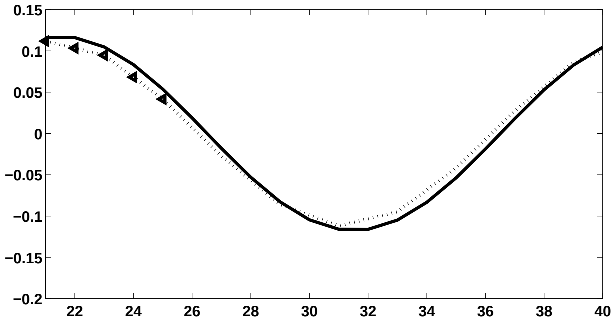
(a) Solution on lower boundary



(b) Error on lower boundary

Figure 1: Comparison of low resolution (dotted line with triangles), standard balanced truncation (dashtione

e



(a) Solution on upper boundary

(b) Error on upper boundary

Figure 2: Comparison of low resolution (dotted line with triangles), standard balanced truncation (dashed line with circles) and α-bounded balanced truncation (solid line with stars) approximations to the buoyancy on the upper boundary

[illegible]

c c). I c a , a da d ba c d ca d (F 4(a),
c c) ca ab a c a d

b [illegible] a [illegible] a [illegible] a [illegible] a [illegible] a [illegible] a [illegible]
[illegible] d . I [illegible] 4D-Va [illegible] d [illegible] ac [illegible] d b [illegible]
a [illegible] a [illegible] a [illegible] b [illegible] c [illegible] a [illegible] d b [illegible] a [illegible] d [illegible] a [illegible]
a d [illegible] c b [illegible] a [illegible] a [illegible] . I
a [illegible] a [illegible] a [illegible] ca [illegible], [illegible] c [illegible] b [illegible] d [illegible] a
a [illegible] a Ga [illegible]-N [illegible] c d . Eac [illegible] a [illegible] d
c a [illegible] a [illegible] a [illegible] a [illegible] b [illegible] b c [illegible] a [illegible] d [illegible] a [illegible],
a [illegible] a [illegible] d [illegible] (TLM). T [illegible] TLM [illegible] d d c d [illegible] a [illegible] a [illegible]
a d [illegible] a b [illegible] ab [illegible] a fi [illegible] d . T [illegible] a [illegible] c [illegible] a
[illegible] d [illegible] a d [illegible] a [illegible] a [illegible] a [illegible] d [illegible] ab [illegible].
U [illegible] a [illegible] TLM [illegible] a [illegible] a d b [illegible] a [illegible] d [illegible] a [illegible] a a [illegible]
[illegible].

I [illegible] a [illegible] a [illegible] d [illegible] a [illegible] d [illegible] d c [illegible] d [illegible] α-
b [illegible] d d ba [illegible] c d [illegible] ca [illegible] a b [illegible] d [illegible] b a [illegible] b [illegible] a [illegible] a-
[illegible] ab [illegible] TLM [illegible] Ga [illegible]-N [illegible] c d . T [illegible] d [illegible]
d c [illegible] c [illegible] c [illegible] a [illegible] d [illegible] a [illegible] a [illegible] TLM [illegible]
[illegible] ca [illegible] a [illegible]. I ca [illegible] b [illegible] a [illegible] d [illegible] d [illegible] d [illegible]
[illegible] b [illegible] ab [illegible] d [illegible] . T [illegible] c [illegible] a
[illegible] ba [illegible] b [illegible] d ca [illegible] b [illegible] d.

T [illegible] d [illegible] d [illegible] c [illegible] a [illegible] a [illegible], [illegible], a d
[illegible] d d [illegible] d [illegible] a [illegible] a [illegible] b [illegible] a [illegible] a [illegible]. H [illegible],
[illegible] b [illegible] a [illegible] d [illegible] ac [illegible] ca [illegible] b
[illegible] K [illegible] b [illegible] ac [illegible] c [illegible] fi d [illegible] c [illegible].

W [illegible] a [illegible] c [illegible] a d [illegible] α-b [illegible] d d ba [illegible] c d [illegible] ca [illegible] d
[illegible] a da d ba [illegible] c d [illegible] ca [illegible] a [illegible] ac [illegible] ab [illegible] a d [illegible]
[illegible] a [illegible] a [illegible] ca [illegible] a 2-d [illegible] a [illegible]
Ead [illegible] d [illegible] T [illegible] Ead [illegible] d [illegible] a [illegible] d [illegible] ba [illegible] c [illegible] c [illegible] ab [illegible],
c [illegible] d [illegible] a [illegible] c a [illegible] a [illegible] d [illegible] d [illegible].
I [illegible] ca [illegible] d [illegible] a [illegible] c [illegible] a [illegible] α-
b [illegible] d d a [illegible] a [illegible] d. I ca [illegible] d [illegible] a [illegible] b [illegible] a [illegible]
[illegible] d [illegible]. T [illegible] d [illegible] a [illegible] a [illegible] b [illegible] a c [illegible]
a d [illegible] b [illegible] da [illegible] a d [illegible] d [illegible] ab [illegible] d [illegible]. I
[illegible] d, [illegible] a [illegible] d [illegible] b [illegible] a [illegible] a [illegible] d [illegible]
a [illegible] d [illegible] a [illegible] a [illegible] a [illegible] a [illegible] c [illegible].

T [illegible] a [illegible] d [illegible] a [illegible] b [illegible] NERC Na [illegible] a [illegible] C [illegible] Ea
Ob [illegible] a [illegible].

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